THE POSSIBILITIES OF UNCERTAINTY ESTIMATION BASED ON THE RESULTS OF MEASUREMENT SYSTEM ANALYSIS

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Abstract

The quality of metallurgical products is affected not only by the quality of the production process, but also by the quality of the measured data through which is the production process controlled. For this reason one of the prerequisites for success in the market is quality of the measurement system. This is the reason why the evaluation of measurement system quality is an integral part of the activities of quality planning and quality control in both the automotive and also in the metallurgical industry. At present the evaluation of the quality of used measurement system is mostly performed by using two approaches respectively methodologies. It is a Measurement System Analysis (MSA) methodology developed by the U.S. automotive industry and VDA 5 methodology developed by the German automotive industry. The main objective of this paper is the analysis of the interrelationships among the most important properties of the measurement system evaluated according to MSA guide and measurement uncertainties evaluated according to VDA 5 guide. The factors that have the greatest impact on the differences in the results obtained by using both methodologies are described.

Keywords:
Measurement system analysis, uncertainties, repeatability, reproducibility, ANOVA

1. INTRODUCTION

The evaluation of the quality of measurement systems used for planning, management and continuous improvement of production processes is one of the important prerequisites for providing quality products in all industrial fields. At present, there are several different approaches which can be used to evaluate this quality. The purpose of this paper is to present two approaches/methodologies that are widely used within the scope of both automotive and metallurgical industries.

1.1 Measurement system analysis

The first presented approach is the measurement system analysis known under the shortcut MSA. An MSA manual describes several basic procedures and methods for the performance of the measurement system analysis. This manual was created by a group of American car manufacturers Chrysler Group LLC, Ford Motor Company and General Motors Corporation and it provides guidance on how to assess the quality of a measurement system based on the evaluation of the most important statistical properties of the measurement system. The manual contains procedures for the evaluation of stability, bias, linearity and repeatability and reproducibility of the measurement system. The most commonly evaluated combined repeatability and reproducibility of measurement system (GRR) uses three methods. They are:

- Range method
- Average and range method
- ANOVA
The use of the simplest range method features a number of disadvantages, which is reflected in its minimum practical utilization and that is why our attention in this article will be focused on the two remaining methods.

1.1.1 Average and range method

Average and range method (A&R) is most commonly used for measurement system repeatability and reproducibility assessment in the practice. Required data are obtained by repeated measurements of product samples realised by various operators. With using defined procedure [1], which includes both numeric and graphical evaluation, repeatability (EV) and reproducibility (AV) are evaluated.

On the basis of EV and AV values it is possible to calculate combined repeatability and reproducibility (GRR) according to the relation:

\[ GRR = \sqrt{(EV)^2 + (AV)^2} \]  

(1)

As criteria of measurement system acceptability the percentage share of GRR in total variation and number of distinct categories (ndc) are used. They are calculated with using relations (2) and (3).

\[ \%GRR = \frac{GRR}{TV} \cdot 100 \]  

(2)

\[ ndc = 1,41 \cdot \frac{PV}{GRR} \]  

(3)

where: \( TV \) – total variation \( (TV = \sqrt{(GRR)^2 + (PV)^2}) \)  

\( PV \) – parts variation

Measurement system is considered as fully acceptable in the cases, when \( \%GRR \) value is lower than 10% and at the same time ndc value is 5 minimally.

1.1.2 ANOVA

The last, fourth edition of the MSA manual lays even more emphasis on the evaluation of repeatability and reproducibility by means of the analysis of variance (ANOVA). In case of this method, the total variability can be divided into equipment variability (EV), operator variability (AV), variability between parts (PV) and the interaction between operators and parts (INT). The evaluation of GRR study using this method makes it possible to acquire more information than in case of the average and range method, as it also provides additional information on how much of the total variability is caused by the interaction among the individual operators and parts [8]. If this interaction is statistically significant, its value is recorded separately and the combined repeatability and reproducibility is calculated as follows:

\[ GRR = \sqrt{(EV)^2 + (AV)^2 + (INT)^2} \]  

(5)

If the interaction is not statistically significant, it is assigned to the value of repeatability. ANOVA method can therefore detect much more accurate estimates of the variances, provided that the measurement errors are normally distributed. This assumption can be verified using suitable graphical tools presented in work [9]. The disadvantage of this method includes more complicated calculations of the individual elements of variability, which is why it is necessary to use a computer during its application.
1.2 Measurement uncertainty

The measurement uncertainty according to VDA 5 manual [2] is a parameter associated with the results of a measurement that characterizes the dispersion of values that could be reasonably attributed to the measured variable. A parameter can be understood as a standard deviation or its specified multiple. The evaluation of uncertainties is performed in the following order:

- standard uncertainty
- combined standard uncertainty
- extended uncertainty of measurement

Standard measurement uncertainty can be evaluated using two methods. The first of them is the A method, which is based on the evaluation of standard deviation of the measured values. If we cannot determine the standard uncertainty using the A method, or its determination by means of this method is not cost effective, the relevant standard uncertainties can be estimated from previous information, i.e., using the B method. The previous information may include, for example, manufacturer’s data, data from calibration sheets or information from previous or older measurements, etc. These two methods should be used to gradually evaluate all the relevant sources of uncertainty. The combined standard uncertainty is marked \( u(y) \) and is calculated from all components of uncertainties identified by the A and B methods based on a mathematical model. If the so-called sensitivity coefficients equal „one”, the combined standard uncertainty is determining using a quadratic sum:

\[
\begin{align*}
  u(y) &= \sqrt{\sum_{i=1}^{n} u(x_i)^2} \\
  \text{(6)}
\end{align*}
\]

The extended uncertainty gives a measure of uncertainty which can set apart the true value from the measured one. The second edition of the VDA 5 manual distinguishes between the \( U_{MS} \) extended measurement uncertainty, evaluated only on the basis of the components of uncertainty related to the measuring instrument, the controlled indicator and the standard, and the extended uncertainty of the measurement process \( U_{MP} \), evaluated on the basis of all relevant uncertainty components. This value is calculated by multiplying the combined measurement uncertainty by the coverage factor \( k \), which determines the scope of the confidence interval.

\[
\begin{align*}
  U_{MP} &= k \cdot u(y) \\
  \text{(7)}
\end{align*}
\]

In practice, the value of \( U_{MS} \) should therefore correspond to the value of measurement repeatability \( EV \) and the value of \( U_{MP} \) should correspond to the value of combined repeatability and reproducibility \( GRR \). \( Q_{MS} \) and \( Q_{MP} \) suitability indicators are introduced to assess the suitability of the measuring system and the measuring process. The percentage expressions of these indicators are:

\[
\begin{align*}
  Q_{MS} &= \frac{2 \cdot U_{MS \_SPECIFICATIONS}}{100} \\
  Q_{MP} &= \frac{2 \cdot U_{MP \_SPECIFICATIONS}}{100} \\
  \text{(8)} \quad \text{(9)}
\end{align*}
\]

The values of these indicators are then compared to the limit values of \( Q_{MS\_max} \) resp. \( Q_{MP\_max} \). The recommended limit value of the measuring systems is \( Q_{MS\_max} = 15\% \) and the recommended limit value of the measurement processes is \( Q_{MP\_max} = 30\% \). The \( Q_{MS\_max} \) and \( Q_{MP\_max} \) limit values may also be determined upon agreement with the customer.
2. EVALUATION OF THE MEASUREMENT UNCERTAINTY BASED ON DATA SETS USED FOR GRR STUDY

The main objective of this work was to analyze the relationships between the measurement system characteristics and the measurement uncertainty. This chapter contains the results of the performed simulations and calculations using real data, as well as the evaluation of the differences in the final values of the individual statistical properties of the measurement system and the measurement uncertainties, while using the procedures specified in both methodologies. The analysis and comparison required the total of 7 data sets from literature [1-7] used to evaluate the repeatability and reproducibility of the measurement system. The results presented in the reference sources, from which the measured data sets had been taken, are incomplete, which is why comprehensive analyses of the measurement repeatability and reproducibility of all of these data sets were performed for the purpose of this chapter. The evaluation of these analyzes was performed in statistical program Minitab 16, using the A&R method, as well as the ANOVA method. The same data sets were subsequently used to evaluate the measurements uncertainty and Q_{MS} and Q_{MP} indicators according to the VDA 5 methodology. Because the data sets structure used to evaluate the GRR analysis is not identical to the standard structure of the measured data sets, which should be used to evaluate the measurements uncertainty, the measurement uncertainty evaluation took advantage of a modified procedure.

2.1 Procedure for evaluation of the measurement uncertainty

The basic condition of mutual comparison of the results obtained using the procedures of both methodologies is the evaluation of the percentage share of EV, AV and GRR values based on the tolerance of the measured quality indicator. Only then it is possible to evaluate the % EV, % AV and % GRR values with the values of Q_{MS} and Q_{MP} indicators. In cases where the used reference sources had not provided the values of the tolerance limits, these limits were calculated in such a way to make the process capable, i.e., the values of C_{p} and C_{pk} indices equal 1.33.

The most important component of the combined measurement uncertainty is usually the repeatability on standard $u_{EVR}$. According to the VDA 5 manual, there should be at least 25 repeated measurements, which are used to calculate the $u_{EVR}$. This value was calculated from actual data sets on the basis of pooled standard deviation of measurements of individual samples by individual operators. This uncertainty includes only the repeatability of measurement and forms the basis of the calculation of the extended measurement uncertainty of the measurement system $U_{MS}$. The next step was to calculate the average values of all measurements of all samples of the given operator. These calculated values were subsequently used to calculate the range of averages ($R_0$). This value is later referred to as the limiting value $a$. Method B was then used to calculate the reproducibility of operators $u_{AV}$, according to the formula (10).

$$u_{AV} = a \cdot b$$  \hspace{1cm} (10)

The value of the distribution factor $b$ depends on the used distribution. If the statistical distribution of boundary error transformation $a$ is unknown, the standard uncertainties are determined using uniform distribution. The next step was to use the $u_{EVR}$ and $u_{AV}$ uncertainties to calculate the combined standard uncertainty according to formula 6. This uncertainty consists of all the components of uncertainties determined by means of the A and B methods. The extended uncertainty of the measurement process $U_{MP}$ was calculated according to formula (9) on the basis of the combined uncertainty. The calculation requires to determine the value of the coverage factor $k$. The value of the coverage coefficient for all data sets was selected as $k = 3$, which corresponds to 99.73 % confidence interval. The final step was to determine a suitability indicator of the measurement process according to the formula (9) [2].
The results of the performed calculations are shown on Figure 1. For the purpose of easier comparison of the final indicators calculated according to the procedures in both methodologies, the values in this figure always represent a percentage expression of difference between the value of the final indicator ($Q_{MP}$) calculated according to the VDA 5 manual and the value of the relevant indicator ($\%GRR$) calculated according to the MSA manual, where value of $\%GRR$ was considered as 100%. The value of 9.54 share for data set 1 and A&R method then means that, in this case, the value of $Q_{MP}$ indicator was about 9.54% higher than $\%GRR$ calculated by A&R method on the basis of the same data set. On the contrary, the value of -32.64 means that the value of $Q_{MP}$ indicator was by 32.64% lower than $\%GRR$ value calculated by ANOVA method on the basis of the same data set.

![Figure 1 Percentage difference between $Q_{MP}$ and $\%GRR$ indicators](image)

The results shown in Figure 1 can lead to several conclusions. Overall, we can say that in most examples, the values of the two most important indicators of $Q_{MP}$ and GRR% are not much different, in case, when the value of GRR% was calculated using the average and range A&R method, since the value of $Q_{MP}$ was significantly lower than the value of GRR% only in one example.

**2.3 The effect of interactions**

The results presented in Figure 1 also clearly show that the differences in the results of the analyses performed by means of both methods in these examples are significantly different. These are data sets 3, 5 and 7. This difference is evident only in the evaluation of GRR using ANOVA method, and it is due to a very low value of the $u_{AV}$ uncertainty component in all three data sets, because the evaluation of GRR using the A&R method shows minimal difference between the monitored values. This disparity of results confirms the conclusions of our previous research, in which we have stressed the effect of the interactions between measured parts and operator on the diversity of GRR study results, achieved using the A&R and ANOVA methods. The influence of interaction is clearly shown on Figure 2, where statistically significant interaction is found in five data sets [8].

**CONCLUSION**

All performed experiments on real data sets show the connection between the VDA 5 and MSA guides. The best agreement was reached in the case of measurement repeatability, where for most examples were the differences between the resulting values small. This fact is especially noticeable in the case when we using ANOVA method, when in five examples were the differences between the values $u_{EVR}$ and EV% almost negligible. Conversely, in the case of measurement reproducibility evaluation, in most cases they are significant differences in the values $\%EV$ evaluated by ANOVA method compared to the values $u_{AV}$
calculated by the procedure in the VDA 5 guide. This situation is caused by the occurrence of a statistically significant interaction between the operator and the measured sample, which can be evaluated only by using the ANOVA method. It can therefore be argued that the most complex results can be achieved using ANOVA method. This fact confirms the general trend of preferring ANOVA method in the procedures described in the two most widely used guides for assessing the quality of the measurement system.

![Graph showing influence of interactions on percentage difference between Q_{MP} and %GRR indicators](image)

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**LITERATURE**


