THE THEORETICAL CALCULATION OF ROLLING PRESSURE MINIMUM

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Abstract
Differential equation describing distribution of rolling pressure in rolling zone was introduced by von Karman. All known solutions for the calculation of rolling pressure are based on von Karman’s differential equation. Work roll is described by the equation of a circle. The Von Karman’s differential equation has not yet any known analytical solution. Up till now, solutions have relied upon the assumption of equation of a circle approximation. Korolev used a polyline to approximate a circle, Tselikov used a line, Sims and Blend & Ford used the equation of a parabola in terms of polar coordinates. Pernis used parabola in terms of the Cartesian coordinates. Curves of the average rolling pressure are the function of a shape factor $l_p/h_{av}$ (projected arc of contact to average of entry and exit thickness). Mapping of the calculated curves shows monotonic growth. Experimental measurements of the average rolling pressure in the surroundings of the point, which has the value of shape factor $l_p/h_{av}=1$, evince local minimum. Tselikov established empirical relation to calculate an average rolling pressure for shape factor $l_p/h_{av}<1$. The paper presents the calculation of local minimum of an average rolling pressure which depends on the position of neutral points in rolling zone. Position of local minimum for average rolling pressure is shown in figures in the paper.

Key words: Flat rolling, Neutral angle, Rolling pressure distribution, Minimum of rolling pressure, Rolling Force.

1. INTRODUCTION
A force equilibrium in a rolling zone at axial rolling is expressed by a differential equation introduced by [1] in the form

$$\frac{d}{dx}(\sigma_x \cdot y) = \sigma_n \cdot (\sin \varphi \pm f \cdot \cos \varphi), \quad (1)$$

where $\sigma_n$ is the direct stress on rolls, $\sigma_x$ represents axial stress in a rolled product and $f$ is the friction coefficient between rolls and rolling material. The variables $x$, $y$ and $\varphi$ represent the coordinates of the roller gripping arc affecting the rolling material. In the differential equation (1) shear stress $\tau$ is expressed by the Coulombo’s equation $\tau = f \cdot \sigma_n$. The stresses $\sigma_n$ and $\sigma_x$ represent the main stresses. Next the plasticity equation is defined by the relationship $\sigma_n - \sigma_x = \sigma_a$. The stress $\sigma_a$ represents the actual deformation resistance (that is the basic deformation resistance $\sigma_p$). In rolling without stabilization $\sigma_a=\text{const}$. The direct stress $\sigma_n$ in equation (1) is transposed to relative direct stress $\bar{\sigma}_n$.

$$\bar{\sigma}_n = \frac{\sigma_n}{\sigma_a}. \quad (2)$$

At given assumption the modified Karman’s differential equation has the form

$$\frac{d\bar{\sigma}_n}{dx} - \frac{1}{y} \frac{dy}{dx} \frac{f}{y} \bar{\sigma}_n = 0. \quad (3)$$

2. DISTRIUTION OF CONTACT PRESSURE

Differential equation (1) that is (3) represents in fact 2 differential equations. The equation with a sign + describes the direct stress $\sigma_{nF}$ in area of forward zone and the equation with the – describes the direct contact stress $\sigma_{nB}$ in area of backward slip. In the solution, submitted by Pernis [6] is the circle arc approximated by a parabola. As a result of the differential equation solution (1), were acquired equations (4) and (5), describing the direct contact stress $\sigma_{nF}$ a $\sigma_{nB}$.

$$
\begin{align*}
\sigma_{nF} &= 2a \left[ \frac{1}{2} + \frac{1}{m} \right] \cdot e^{m} - \frac{u}{m} \cdot \left( \frac{1}{m^2} \right), \\
\sigma_{nB} &= 2a \left[ \frac{1}{2} - \left( \frac{u_0}{m} \cdot \frac{1}{m^2} \right) \right] \cdot e^{m(u-u_0)} + \frac{u}{m} \cdot \left( \frac{1}{m^2} \right),
\end{align*}
$$

(4)

(5)

where $u$ represents angle coordinate and $u_0$ its maximum value.

Fig. 1 Distribution of a relative contact stress in a rolling zone: a, $u_n>0$, b, $u_n=0$, c, for $u_n<0$, d, representation of position N at negation $u_n$. 

a, $D=210\,\text{mm}, h=7\,\text{mm}$ $E=30\%\quad m=0,111\quad f=0,12\quad u_n=0,145$

b, $D=210\,\text{mm}, h=7\,\text{mm}$ $E=30\%\quad m=0,614\quad f_{\text{nm}}=0,066\quad u_n=0$

c, $D=210\,\text{mm}, h=7\,\text{mm}$ $E=30\%\quad m=0,185\quad f=0,02\quad u_n=0$

d, $D=210\,\text{mm}, h=7\,\text{mm}$ $E=30\%\quad m=0,370\quad f=0,04\quad x_n=-4,93\,\text{mm}$ $N\quad [-4,93;0,97]$
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\[ u = \arctg \left( \frac{e}{\sqrt{1-e^2}} \right), \quad u_0 = \arctg \left( \frac{e}{\sqrt{1-e^2}} \right), \quad \text{(6), (7)} \]

while \( \varepsilon \) represents relative deformation at one transition and \( l_p \) represents the gripping arc length. The constant \( m \) results from the differential equation solution (1)

\[ m = \frac{-2f \cdot l_p}{\sqrt{\Delta h \cdot h_1}}, \quad \text{(8)} \]

where \( \Delta h \) is the absolute reduction and \( h_1 \) represents material thickness at roller output. In rolling zone, where valid \( \sigma_{nF} = \sigma_{nB} \) exists a neutral point \( N \). The visualization of equations (4) (5) is presented in Fig. 1. The representation is accomplished for roller average \( D=210 \) mm, entrance thickness \( h_0=7 \) mm and reduction \( \varepsilon =30 \% \), used friction coefficient \( f \) is depicted in the graph. In Fig. 1a the point of curve insertion represents the neutral point \( N \). In case, that it reduces the friction coefficient, the neutral point moves in direction of roller. For friction coefficient \( f=0,066 \) the neutral point \( N \) is found in the last point of material contact with the rollers, see Fig. 1b. Another reduction of the friction coefficient means “curve breakdown”, see Fig. 1c. While studying the distribution curves for contact stress distribution outside the rolling zone, it was demonstrated that the point of insertion of curves exists. See Fig. 1d. For all graphs introduced in Fig. 1 is the constant ratio of roller radius \( R \) for output thickness \( R/h_1=21,4 \). To create a model of the shift of a neutral point \( N \) into a negative coordinate \( x \) it is possible to achieve also at a constant friction coefficient, with the decrease of a ratio \( R/h_1 \). An angle coordinate \( u_0 \) of a neutral point \( N \) is derived from a comparison of equation (4) a (5) \( \sigma_{nF} = \sigma_{nB} \). Solitary value \( u_0 \) is not possible to express by an implicit function, but in the form of an explicit function \( F(\varepsilon, m, u_0) = 0 \), equation (9). That is why for the determination of a value \( u_0 \) defining the position of a neutral point \( N \) was by numerical methods worked up a graph submitted in Fig. 2. From graph it is evident, that for \( m>4 \) is the angle coordinate of the neutral angle \( u_0 \) dependent only on the relative deformation.

\[ \frac{1}{2} \left( \frac{m^2}{2} - mu_0 +1 \right) \cdot e^{m(u_0-u_n)} - \frac{1}{2} \left( \frac{m^2}{2} +1 \right) \cdot e^m u_n + mu_n = 0. \quad \text{(9)} \]

3. **MEAN VALUE OF CONTACT PRESSURE**

Equations (4) and (5) describe the distribution of the direct contact stress in the rolling zone. According to Fig. 3 the stress area of normal pressure in the rolling zone is circumcised by points OANBGO. The length of the rolling zone represents equivalent angle coordinate \( u_0 \). The mean value of contact pressure is
determined by comparison of stress positions of surfaces OANBGO and OEFGO. Concrete calculation of the mean value of contact pressure \( \sigma_{\text{av}} \) is derived from equation (10)

\[
\sigma_{\text{av}} = \frac{1}{u_0} \left[ \int_0^{u_n} \sigma_{\text{BF}}(u) \cdot du + \int_{u_n}^{u_0} \sigma_{\text{BN}}(u) \cdot du \right].
\]  

(10)

After calculation of indicated integrals, an equation is acquired, suitable for calculation of the mean value of contact pressure \( \sigma_{\text{av}} \)

\[
\sigma_{\text{av}} = \sigma_a \cdot \frac{2}{\mu u_0} \left[ \left(1 + \frac{2}{m^2}\right) \left(e^{\mu u_0} - 1\right) + \frac{u_0^2}{2} - u_n \cdot \left(u_n + \frac{2}{m}\right) \right].
\]  

(11)

\[
\sigma = \frac{\sigma_{\text{av}}}{\sigma_a}.
\]  

(12)

From the equation (11) defined equation \( \sigma \) – function has the form given by an equation (13). Visualization \( \sigma \) – of function in dependence on ratio \( f/\mu \) for the friction coefficient \( f=0.2 \) and \( f=0.4 \) is given in Fig. 4.

\[
\sigma = \frac{2}{\mu u_0} \left[ \left(1 + \frac{2}{m^2}\right) \left(e^{\mu u_0} - 1\right) + \frac{u_0^2}{2} - u_n \cdot \left(u_n + \frac{2}{m}\right) \right].
\]  

(13)

\( \sigma \) – function shows the minimum, which is in the value \( f/\mu =1 \), but is dependent also on relative deformation \( \varepsilon \) and the friction coefficient \( f \). The value of \( \sigma \) – function in minimum can acquire a value less than 1 (\( \sigma <1 \)). In case that from an equation, (9) the calculated angle coordinate \( u_n \) of the neutral point N is negative, for calculation of \( \sigma \) – function an equation (13) is used with the value \( u_n=0 \). For this value, the equation (13) is reduced to relationship
\[ \bar{\sigma} = \frac{u_0}{m} \]  \hspace{1cm} (14)

The equation (14) represented in Fig. 4 describes part of curves, which are to the left of minimum (the decreasing part of the function) and equation (13) describes part of curves, which are to the right of minimum (the increasing part of function). For the calculation of minimum position was established an empirical function from which is calculated the constant value \( m_0 \) with precision \( m_0 \pm 0.001 \)

\[ m_0 = \left( 1.998460 + 0.797608 \cdot \varepsilon - 0.371761 \cdot \varepsilon^2 + 2.591024 \cdot \varepsilon^3 \right) \frac{\sqrt{\alpha(1-\varepsilon)}}{2-\varepsilon}. \]  \hspace{1cm} (15)

The pertaining ratio \( (l_p/h_{av})_0 \) in which the minimum exists has value

\[ \left( \frac{l_p}{h_{av}} \right)_0 = \frac{\sqrt{\alpha(1-\varepsilon)}}{2-\varepsilon} \cdot \frac{m_0}{f}. \]  \hspace{1cm} (16)

For calculation of relative stress in the decreasing part of \( \bar{\sigma} \) function is usable the equation

\[ \bar{\sigma} = 0.5223 \frac{\varepsilon}{f} \left( \frac{l_p}{h_{av}} \right)^{-1}. \]  \hspace{1cm} (17)

Tselikov [3] for the ratio \( l_p/h_{av} \leq 1 \) for calculation of direct stress establishes a simple empirical relationship

\[ \bar{\sigma} = \left( \frac{l_p}{h_{av}} \right)^{-0.4}. \]  \hspace{1cm} (18)

**Fig. 4** The effect of deformation on minimum position \( \bar{\sigma}_{min} \): a, \( f=0.2 \)  b, \( f=0.4 \)
Celikov’s relationship doesn’t take to account the size of deformation and the friction coefficient. The experimentally found values of $\overline{\sigma}$ – function stated in Fig. 5 confirm a marked dispersion of measured points (unstated $\varepsilon$ and $f$).

![Fig. 5 An experimentally values of $\overline{\sigma}$ – function [8]](image)

4. CONCLUSION
The submitted contribution suggests a new view on the theory of longitudinal rolling and the calculation of direct rolling pressure minimum. The minimum position apart from the ratio $l_p/h_{av}$ is dependent also on the relative deformation and friction coefficient. Instead of using Celikov’s equation (18) the use of equation (17) is recommended by authors.

LITERATURE